Core 4 Jan 2006

1) $\dfrac{x^2(x-3)}{(x+3)(x-3)} = \dfrac{x^2}{x+3}$ (3)

2) $\sin y = xy + x^2$ (3)

$\cos y \dfrac{dy}{dx} = x\dfrac{dy}{dx} + y\dfrac{dx}{dx} + 2x\dfrac{dx}{dx}$

$\dfrac{dy}{dx}(\cos y - x) = y + 2x$

$\dfrac{dy}{dx} = \dfrac{y+2x}{\cos y - x}$ (2)

3)

$$\begin{array}{r} 3x+4 \\ x^2-2x+5\,\big)\overline{3x^3-2x^2+x+7} \\ \underline{3x^3-6x^2+15x} \\ +4x^2 \cancel{-14x} +7 \\ \underline{4x^2-8x+20} \\ -6x-13 \end{array}$$

1, 1, 1, 1 (4)

quotient $= 3x+4$

$R = -6x-13$

ii) If no remainder then $+6x+13$ to qu.

$3x^3-2x^2+7x+20$

$a = 7$ $b = 20$ (2)

4) $I = \int x\sec^2 x\, dx$ $u = x$ $\dfrac{du}{dx} = 1$

$\dfrac{dv}{dx} = \sec^2 x$ 1

$v = \tan x$

(4)

$I = x\tan x - \int \tan x\, dx$ 1

$= x\tan x - \ln\sec x + C$

1

ii) $1+\tan^2 x = \sec^2 x$ 1

$\int x\tan^2 x\, dx = \int x(\sec^2 x - 1)$

$= \int x\sec^2 x - x$

$= x\tan x - \ln\sec x - \dfrac{x^2}{2} + C$

2

(3)

5) $x = t^2$ $y = 2t$

$\dfrac{dx}{dt} = 2t$ $\dfrac{dy}{dt} = 2$

i) $\dfrac{dy}{dx} = 2 \times \dfrac{1}{2t} = \dfrac{1}{t}$ (2)

ii) $y - 2p = \dfrac{1}{p}(x - p^2)$ as $t = p$

$\times p$ $yp - 2p^2 = x - p^2$ (2)

$py = x + p^2$ QED

iii) tan at $(9,6)$ $p = 3$ (1)

$3y = x + 9$ ①

tan at $(25,-10)$ $p = -5$ (1)

$-5y = x + 25$ ②

solving ① + ②

$x = -15$ $y = -2$ (2)

6) $x = \sin^2\theta = (\sin\theta)^2$

$\dfrac{dx}{d\theta} = 2\sin\theta\cos\theta$ (1)

$\int\sqrt{\dfrac{x}{1-x}}\, dx = \int\sqrt{\dfrac{\sin^2\theta}{1-\sin^2\theta}} \times 2\sin\theta\cos\theta\, d\theta$ (2)

$= \int\sqrt{\dfrac{s^2}{c^2}} \times 2sc\, d\theta$

$= \int 2\sin^2\theta\, d\theta$ (1)

but $\cos 2\theta = 1 - 2\sin^2\theta$

$\int 2\sin^2\theta = \int 1 - \cos 2\theta$ (1)

$= \theta - \dfrac{1}{2}\sin 2\theta$ (1)

$x = 1$ $\theta = \dfrac{\pi}{2}$ (1)

$x = 0$ $\theta = 0$ (1)

$A\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{2}$ $A(0) = 0$

$I = \dfrac{\pi}{2}$ (1)

7) $\frac{11+8x}{(2-x)(1+x)^2} = \frac{A}{2-x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$ (2)

$11+8x = A(1+x)^2 + B(2-x)(1+x) + C(2-x)$

$x=2$ $27 = 9A$ $A=3$ (1)

$x=-1$ $3 = 3C$ $C=1$ (1)

$x=0$ $11 = A+2B+2C$

$B=3$ (1)

$3(2-x)^{-1} = 3\times 2^{-1}\left(1-\frac{x}{2}\right)^{-1}$

$= \frac{3}{2}\left(1-\frac{x}{2}\right)^{-1}$

$= \frac{3}{2}\left(1+\frac{x}{2}+\frac{(-1)(-2)}{2!}\left(-\frac{x}{2}\right)^2\right)$

$= \frac{3}{2}\left(1+\frac{x}{2}+\frac{x^2}{4}\right.$ (2)

$3(1+x)^{-1} = 3\left(1-x+\frac{(-1)(-2)}{2!}x^2\right)$

$= 3(1-x+x^2)$ (1)

$1(1+x)^{-2} = 1-2x+\frac{(-2)(-3)}{2!}x^2$

$= 1-2x+3x^2$ (1)

adding all 3 answers

$\frac{3}{2}+\frac{3}{4}x+\frac{3}{8}x^2+3-3x+3x^2$

$+1-2x+3x^2$

$= \frac{11}{2}-\frac{17}{4}x+\frac{51}{8}x^2$ (1)

8) $\int y-3\,dy = \int 2-x\,dx$ (1)

$\frac{y^2}{2}-3y = 2x-\frac{x^2}{2}+C$ (2)

$y=4$ $x=5$ (1)

$8-12 = 10-\frac{25}{2}+C$ $C=-\frac{3}{2}$

$\frac{y^2}{2}-3y = -\frac{1}{2}x^2+2x-\frac{3}{2}$ (1)

ii) x by 2 + rearrange

$x^2+y^2-4x-6y+3=0$

$(x-2)^2-4+(y-3)^2-9+3=0$ (3)

$(x-2)^2+(y-3)^2=10$

iii) circle centre $(2,3)$ rad $\sqrt{10}$

9) dir vectors $\begin{pmatrix}-8\\1\\-2\end{pmatrix}^{a}$ + $\begin{pmatrix}-9\\2\\-5\end{pmatrix}^{b}$

$a.b = 72+2+10 = 84$ (2)

$\cos\theta = \frac{84}{|a||b|} = \frac{84}{\sqrt{69}\sqrt{110}}$ (1) (1)

$\theta = 15.4^\circ$ (1)

ii) at intersection

$4-8t = -2-9s$ x's (1)

$2+t = a+2s$ y's (1)

$-6-2t = -2-5s$ z's (1)

solving x's + z's

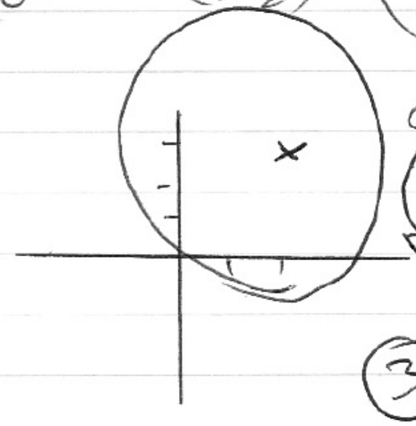
$t=3$ + $s=2$ (2)

subst in y eqn $a=1$ (1)

subst for t gives

$r = \begin{pmatrix}4\\2\\6\end{pmatrix}+3\begin{pmatrix}-8\\1\\-2\end{pmatrix} = \begin{pmatrix}-20\\5\\-12\end{pmatrix}$ (2)

8 iii)



circle $(2,3)$ rad $\sqrt{10}$

(3)